




Neural Network Potentials for Atomistic Simulations




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Overview



Topic of this talk

Machine Learning as a tool to extend the time and length scales of ab initio molecular dynamics

Goal 1: General method for all types of systems

Goal 2: Quality of reference method (here DFT)

Goal 3: Answer questions that cannot be answered with AIMD directly

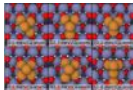
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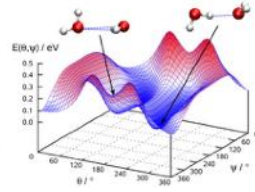
Introduction RUB

Goal:
Atomic-level understanding of complex systems in chemistry and materials science
 ⇒ Predictive computer simulations with first-principles quality

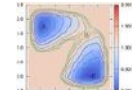
Energy
global and local minima




Central Role:
Potential Energy Surface



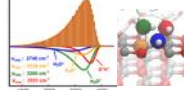
Reactions
barriers / transition states



Forces
dynamics, free energies



Vibrations
properties, analysis



The accuracy of the obtained results depends on the quality of the atomic interactions


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Potentials for Different Length and Time Scales RUB

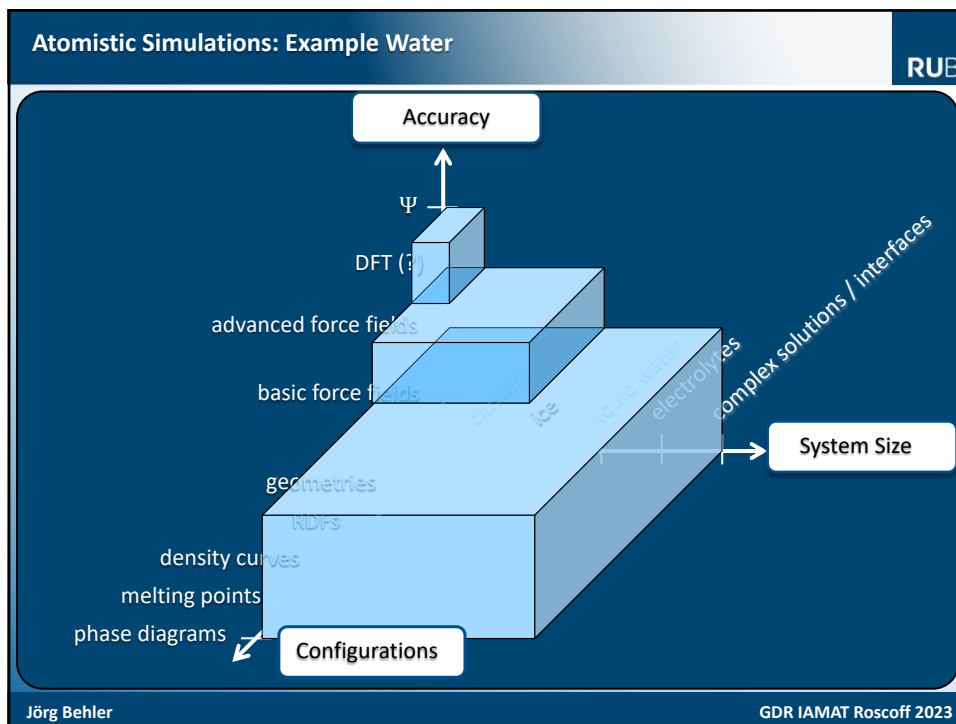
⇒ PES is the central quantity for atomistic simulations

Method	Atoms	Simulation Time
CI, CC, MP2, MP4, ...	10 - 100	0
Density Functional Theory	100 - 1000	100 ps
Semiempirical Methods Tight Binding	1000 - 10 000	1 ns
„Reactive Potentials“ EAM, Tersoff, ...	1000 - 100 000	10 ns
Classical Force Fields (LJ, harmonic, Coulomb)	1 000 000	1 ms

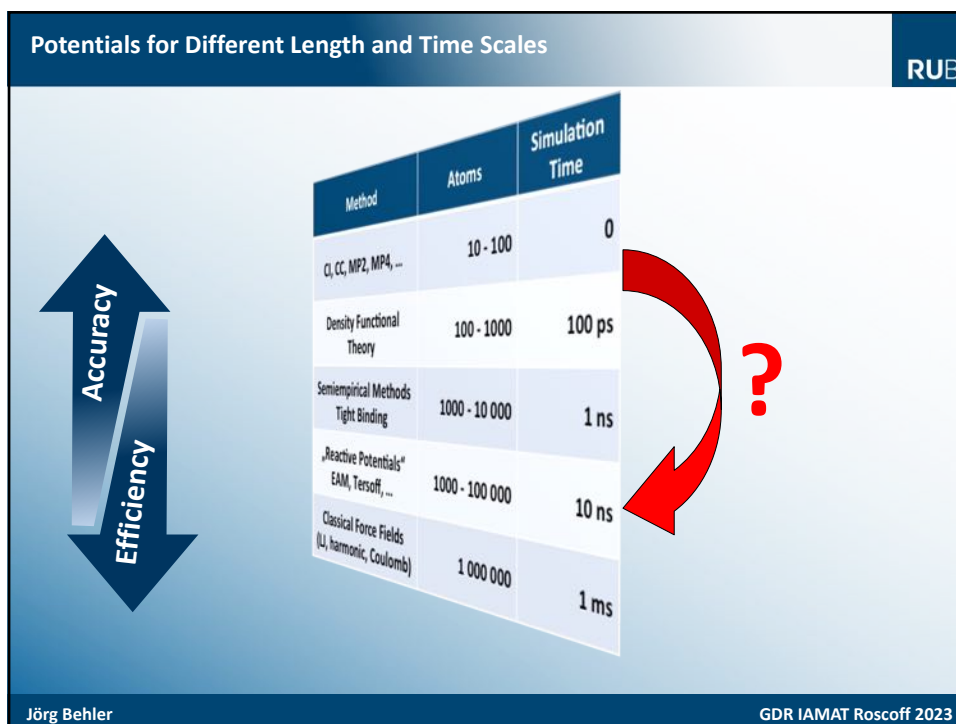


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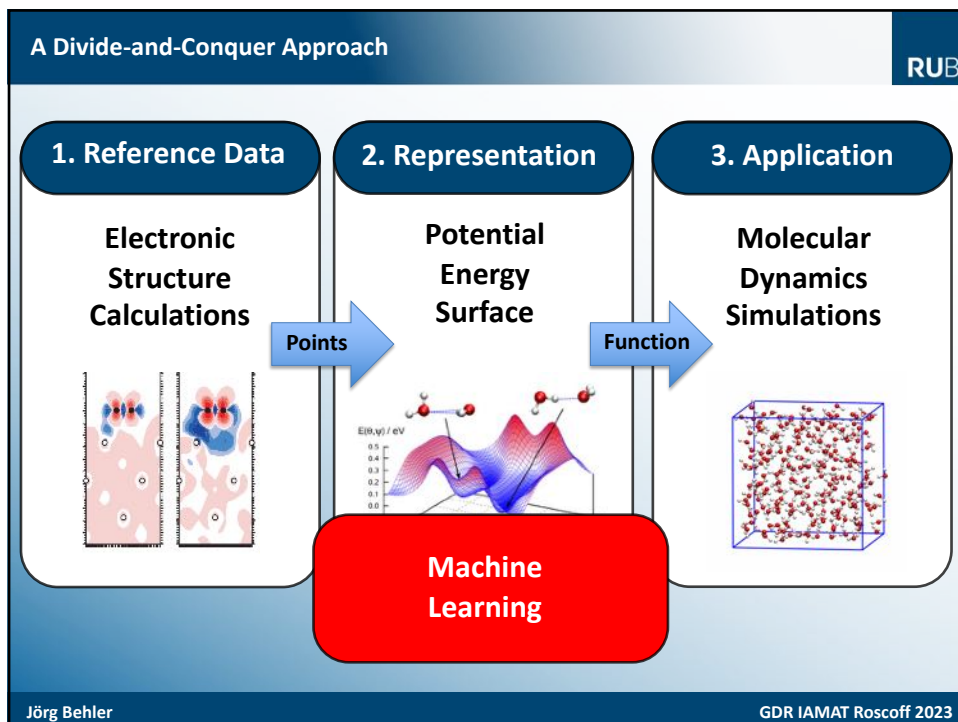
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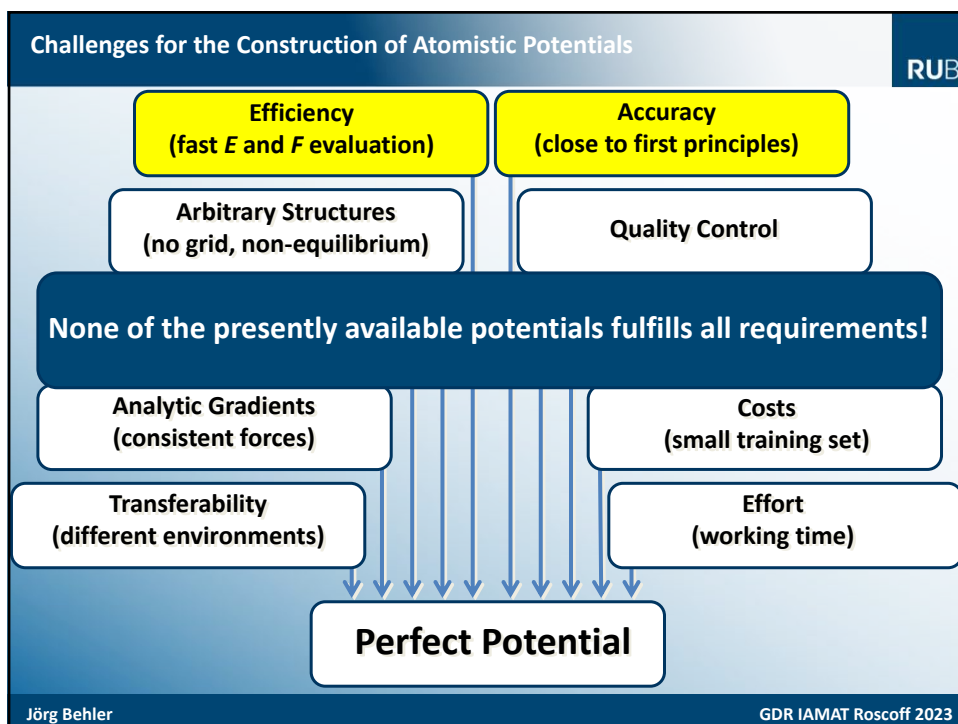
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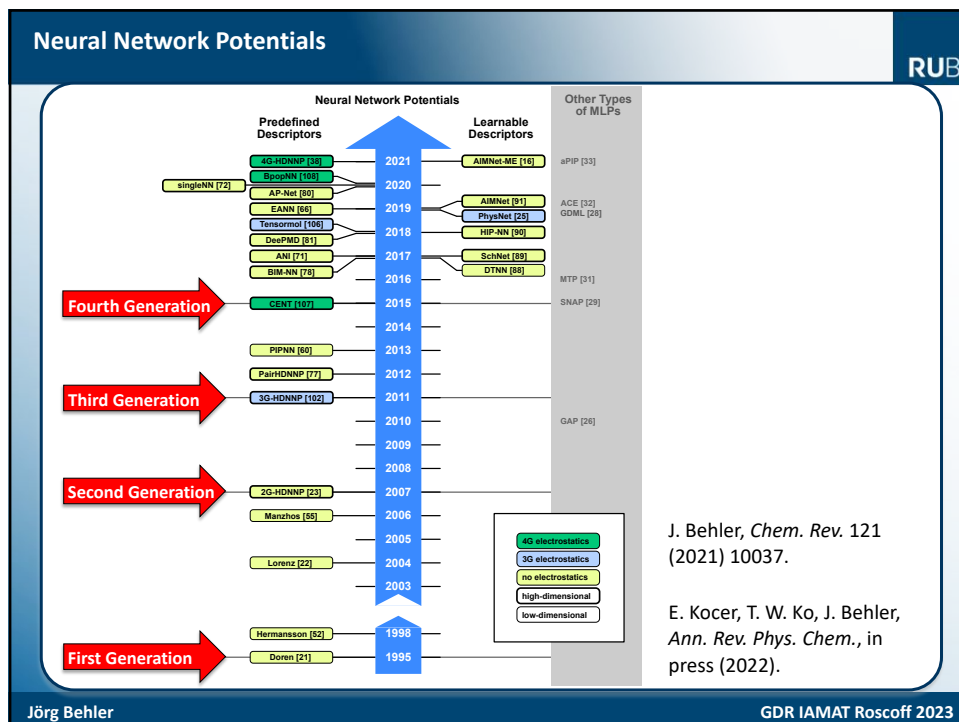
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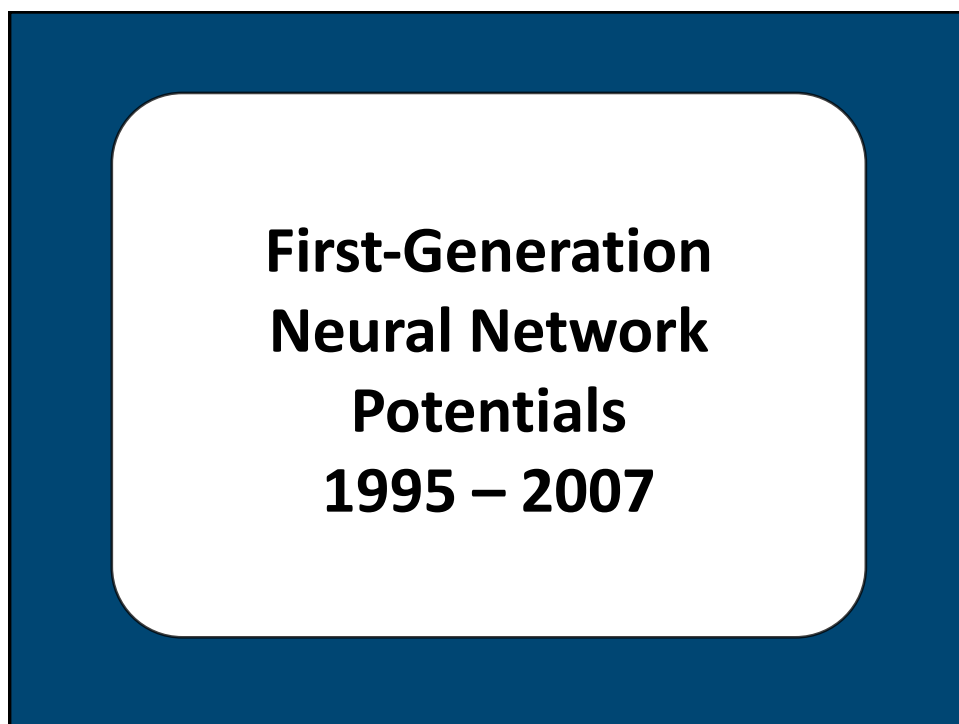
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Machine Learning Potentials RUB

1995 2000 2005 2010 2015 2020

First Machine Learning Potential = First Neural Network Potential

T.B. Blank, S.D. Brown, A.W. Calhoun, and D.J. Doren, *J. Chem. Phys.* **103** (1995) 4129.

Input Layer Hidden Layer 1 Hidden Layer 2 Output Layer

Feed-forward neural network

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Activation Functions: Examples RUB

Activation functions enable the fitting of general nonlinear functions.

Activation functions

- converge for very small and very large arguments
- have a nonlinear shape for intermediate values

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Neural Network Potentials: Activation Functions **RUB**

Basic functional element of the NN: $f(x) = c \cdot \tanh(a \cdot x + b) + d$

(a) $a=3, b=0, c=1, d=0$

(b) $a=1, b=2, c=1, d=0$

(c) $a=1, b=0, c=2, d=0$

(d) $a=1, b=0, c=1, d=1$

⇒ very simple but flexible

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Neural Network Potentials: Activation Functions **RUB**

Basic functional element of the NN: $f(x) = c \cdot \tanh(a \cdot x + b) + d$

(a)

(b)

(c)

(d)

⇒ very simple but flexible

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Model Potential: Harmonic Oscillator

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Goal: Represent potential of the 1D harmonic oscillator in the interval [-3, 3]

• just two activation functions provide a good approximation
• further improvement possible by adding more functions

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Machine Learning Potentials

RUB

1995 2000 2005 2010 2015 2020

First-Generation MLPs

- about 30 papers (1995 – 2007) from about 10 groups
- all methods in the first decade are based on neural networks

Examples:

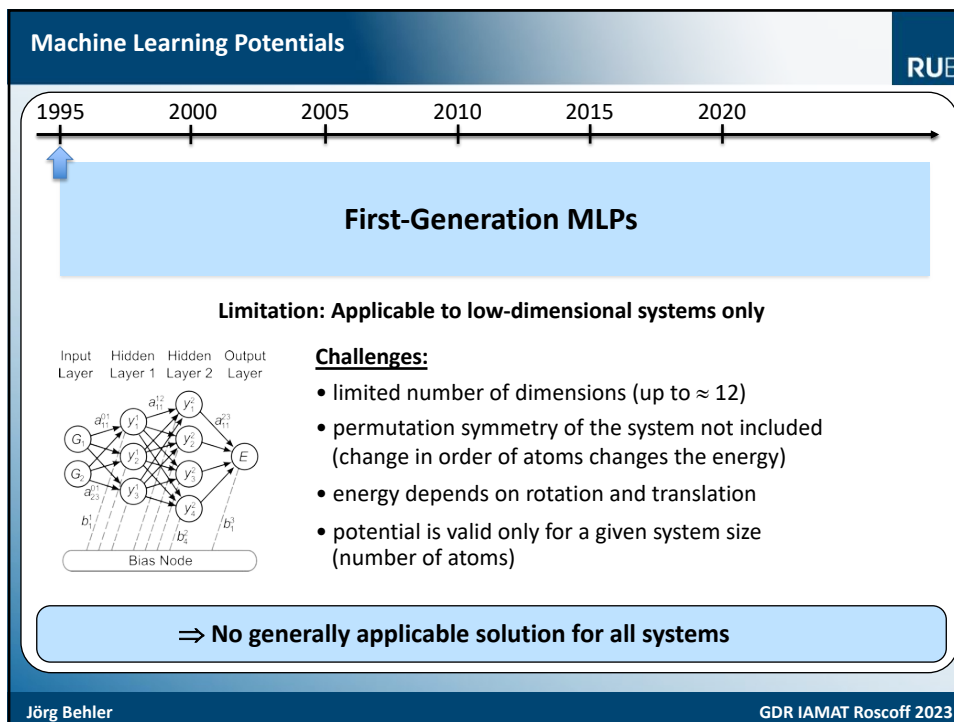
T.B. Blank, S.D. Brown, A.W. Calhoun, and D.J. Doren, *J. Chem. Phys.* **103** (1995) 4129.
 H. Gassner, M. Probst, A. Lauenstein, K. Hermansson, *J. Phys. Chem. A* **102** (1998) 4596.
 S. Lorenz, A. Groß, M. Scheffler, *Chem. Phys. Lett.* **395** (2004) 210.
 S. Manzhos, T. Carrington, Jr., *J. Chem. Phys.* **125** (2006) 194105.
 J. Behler, S. Lorenz, K. Reuter, *J. Chem. Phys.* **127** (2007) 014705.

Focus:
Training
Symmetry
Surfaces
Spectroscopy
Surface Symmetry

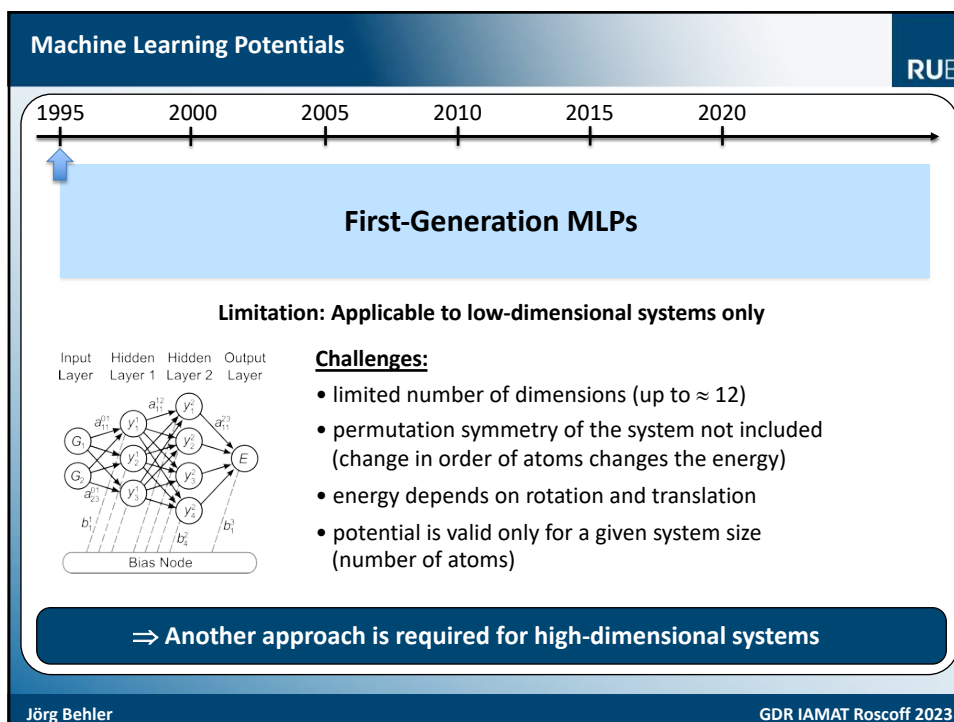
⇒ Basic ideas and key concepts are well established

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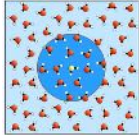
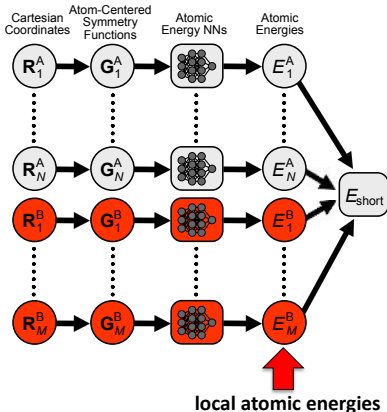
Second-Generation Neural Network Potentials: Locality

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High-Dimensional Neural Network Potentials RUB

3 Steps:

- Total energy is the sum of atomic energies

$$E = \sum_i E_i$$
- Atomic energies depend on local environments
⇒ cutoff radius
 
- Description of local atomic environments by many-body atom-centered symmetry functions
⇒ structural fingerprints
(invariances: rotation, translation, permutation)
 

local atomic energies

⇒ applicable to thousands of atoms

J. Behler, M. Parrinello, *Phys. Rev. Lett.* **98** (2007) 146401.
 J. Behler, *J. Chem. Phys.* **134** (2011) 074106.
 J. Behler, *Angew. Chem. Int. Ed.* **56** (2017) 12828.

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How do the symmetry functions work?

Case 1: Permutation:

Case 2: Rotation:

Case 3: Translation:

Case 4: Symmetry elements:

Example environment:

Symmetry function vector of central atom

$$\begin{pmatrix} 1.223 \\ 1.442 \\ \vdots \\ 2.332 \end{pmatrix}$$

Numerically identical

Atomic NN

\Rightarrow Same property value (e.g. energy)

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Symmetry Functions: Cutoff Function

Cutoff Function

- decays to zero in value and slope at R_c
- reflects decreasing chemical interaction
- central component of all symmetry functions
- R_c is increased until potential converges

$$f_c(R_{ij}) = \begin{cases} \frac{1}{2} \left[\cos\left(\frac{\pi R_{ij}}{R_c}\right) + 1 \right] & \text{for } R_{ij} < R_c \\ 0 & \text{for } R_{ij} > R_c \end{cases}$$

Typical cutoff radius: $R_c = 6 - 8 \text{ \AA}$

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Symmetry Functions: Radial Functions

- decay with increasing distance \Rightarrow Gaussians
- summation over all neighbors
- many-body term, interpretation as coordination number
- one-to-one correspondence between function value and R_{ij}

$$G_i^{rad} = \sum_j e^{-\eta(R_{ij}-R_s)^2} f_c(R_{ij})$$

Set of radial functions:
„Radial Fingerprint“

J. Behler, *J. Chem. Phys.* **134** (2011) 074106.

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RUB

Symmetry Functions

Angular symmetry functions

$$G_i^\mu = 2^{1-\xi} \sum_{j,k \neq i} (1 + \lambda \cos \theta_{ijk})^\xi \cdot e^{-\eta(R_{ij}^2 + R_{ik}^2 + R_{jk}^2)} f_c(R_{ij}) f_c(R_{ik}) f_c(R_{jk})$$

\Rightarrow Several angular functions are used

- all symmetry functions are many-body terms
- typical number: about 10 radial and 40 angular functions

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Symmetry Functions: Combinatorial Growth

RUB

For an N element system, there are N separate atomic NN types
 \Rightarrow no significant increase in complexity

For each radial function there are N functions for the possible neighboring elements
 \Rightarrow increase in complexity

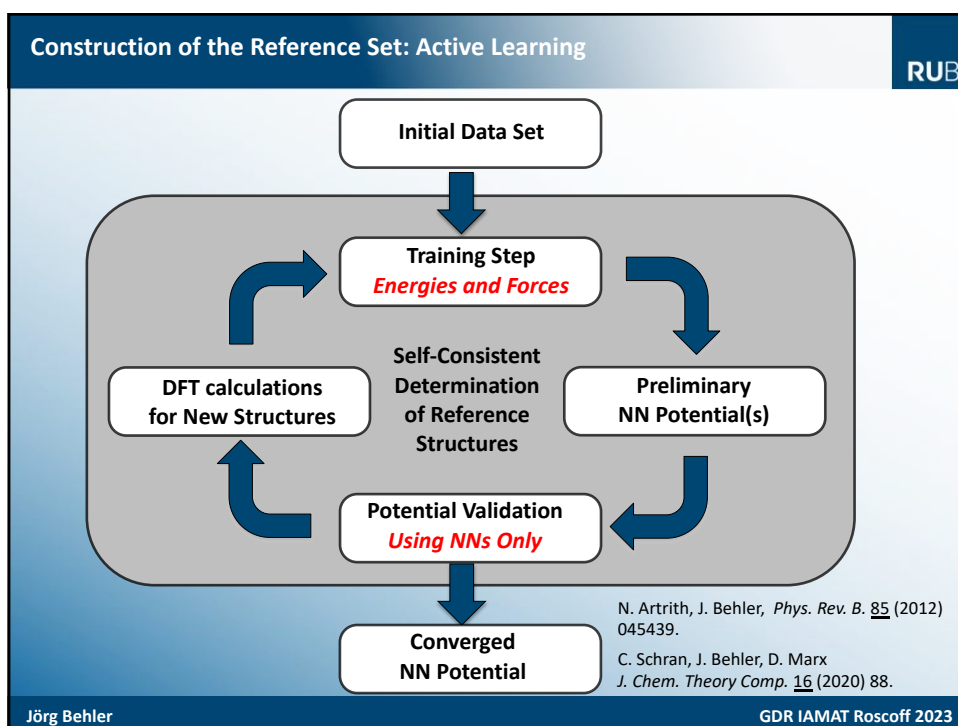
For each angular function there are $N(N+1)/2$ functions for the neighboring elements
 \Rightarrow strong increase in complexity

Elements	atomic NNs	multiplier radial/angular symmetry functions	typical total number of symmetry functions
1	1	1 / 1	6 + 25
2	2	2 / 3	12 + 75
3	3	3 / 6	18 + 150
4	4	4 / 10	24 + 250

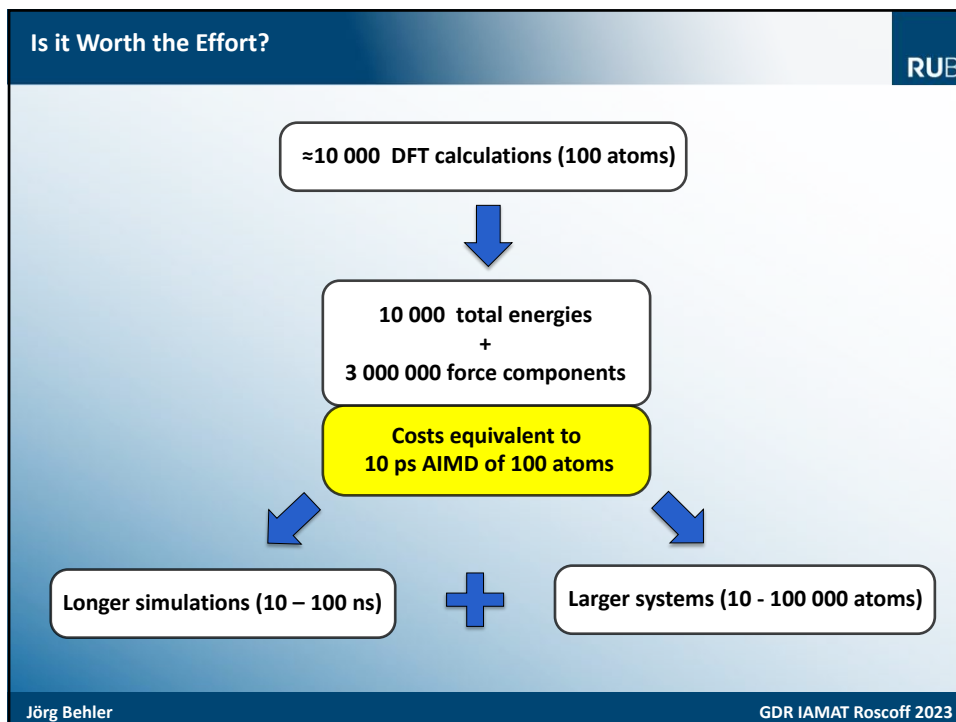
Often, a reduction is possible, if only some compositions or structures are relevant

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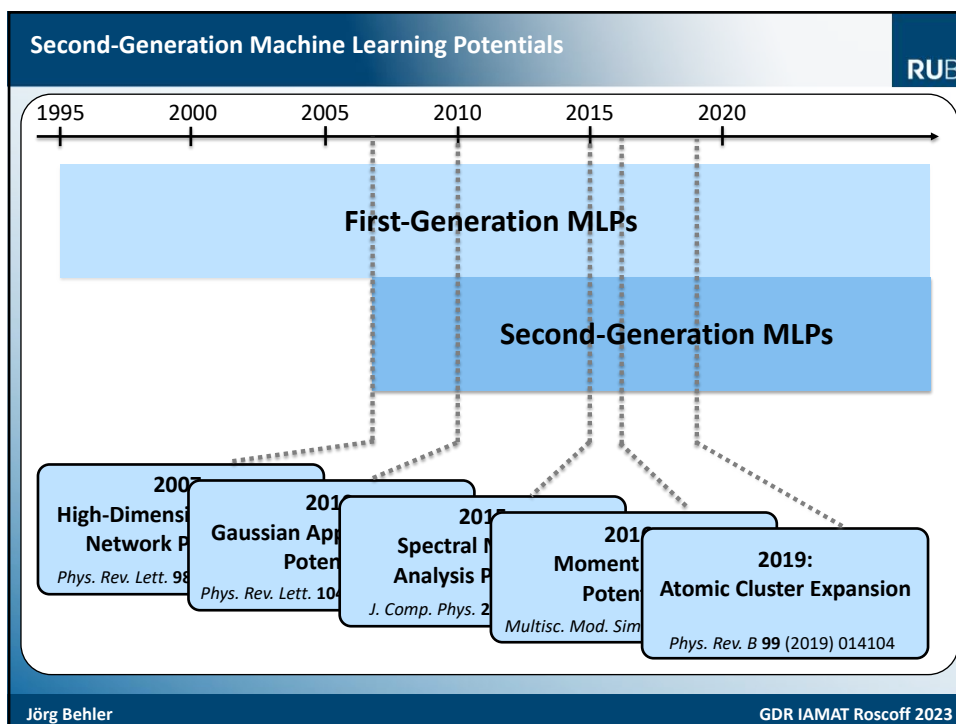
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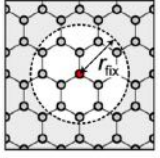
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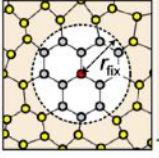
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Interaction Range RUB

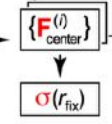
Locality Test



Define fixed sphere around one atom

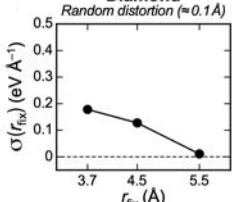


Distort atoms outside r_{fix} (several copies)



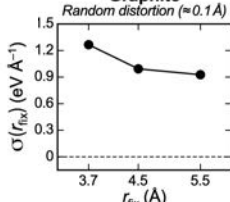
Estimate locality

Diamond
Random distortion ($\approx 0.1 \text{ \AA}$)



$r_{\text{fix}} (\text{\AA})$	$\sigma(r_{\text{fix}}) (\text{eV \AA}^{-1})$
3.7	0.18
4.5	0.12
5.5	0.02

Graphite
Random distortion ($\approx 0.1 \text{ \AA}$)



$r_{\text{fix}} (\text{\AA})$	$\sigma(r_{\text{fix}}) (\text{eV \AA}^{-1})$
3.7	1.2
4.5	1.0
5.5	0.9

V. L. Deringer and G. Csányi, Phys. Rev. B **95** (2017) 094203.

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Interaction Range RUB

A Hessian-Based Analytic Locality Test

Hessian $H_{A_\alpha B_\beta} = \frac{\partial^2 E}{\partial A_\alpha \partial B_\beta} = -\frac{\partial f_{B_\beta}}{\partial A_\alpha} = -\frac{\partial f_{A_\alpha}}{\partial B_\beta} \quad \alpha, \beta = \{x, y, z\}$

		1			2			3			4		
		x	y	z	x	y	z	x	y	z	x	y	z
A	α	x	y	z	x	y	z	x	y	z	x	y	z
	1	y											
	2	y											
	3	y											
4	y												

Hessian submatrix norm

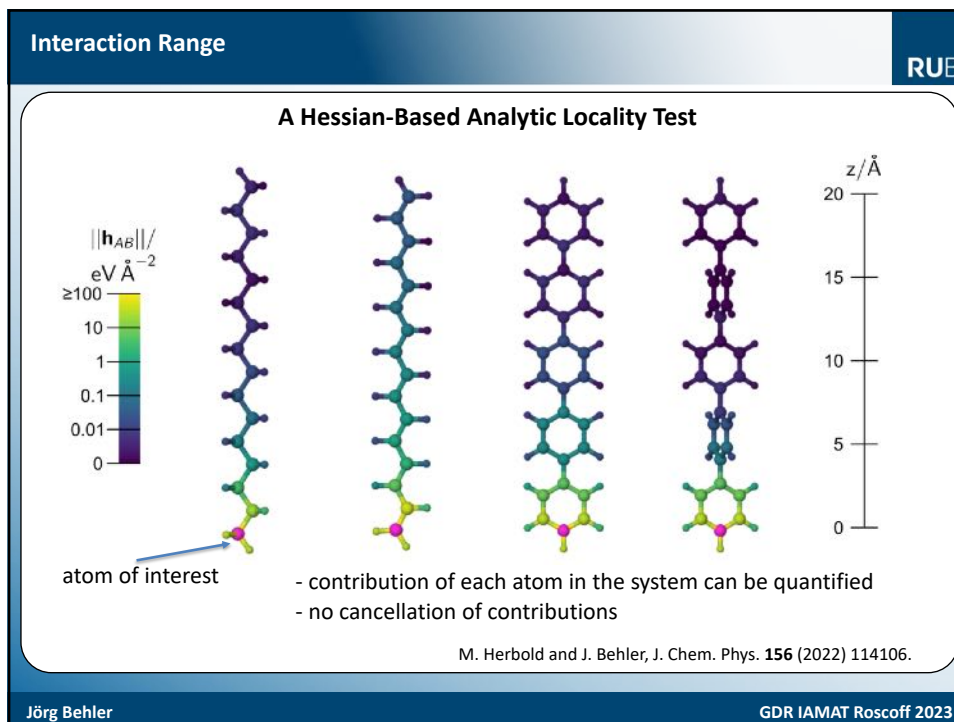
$$\|\mathbf{h}_{AB}\| = \sqrt{\sum_{\alpha=x,y,z} \sum_{\beta=x,y,z} h_{A_\alpha B_\beta}^2}$$

Dependence of force on each individual atom in the system can be quantified

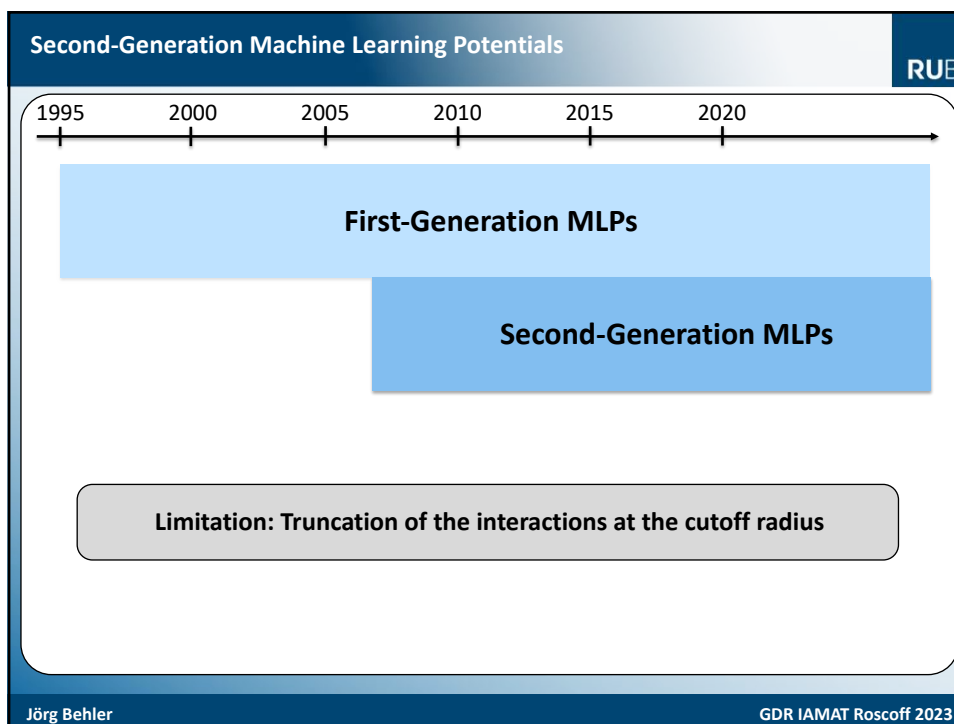
M. Herbold and J. Behler, J. Chem. Phys. **156** (2022) 114106.

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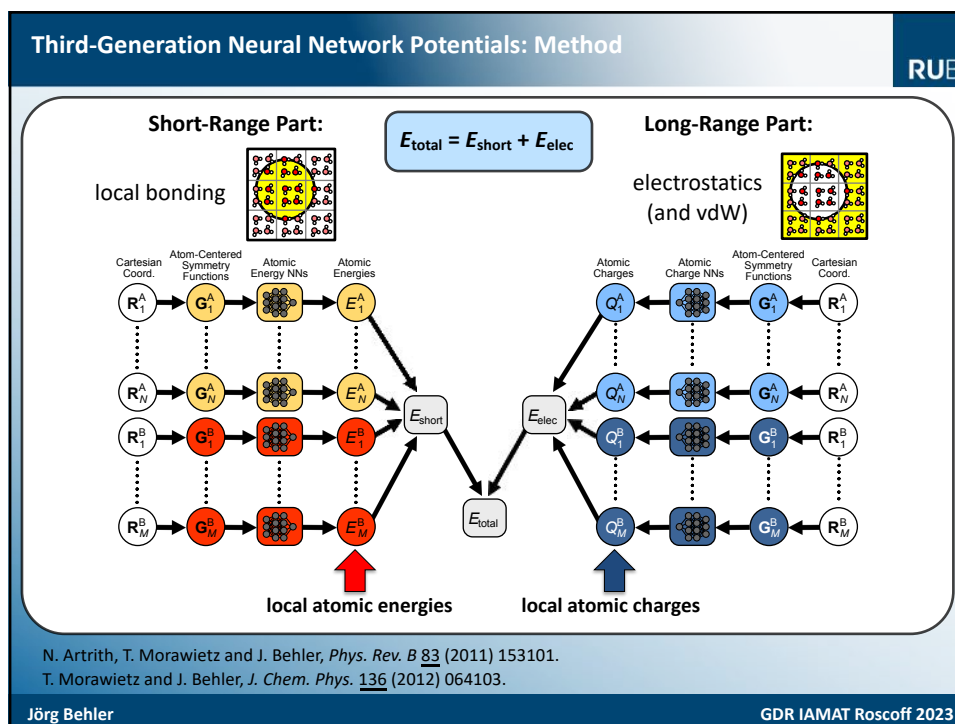
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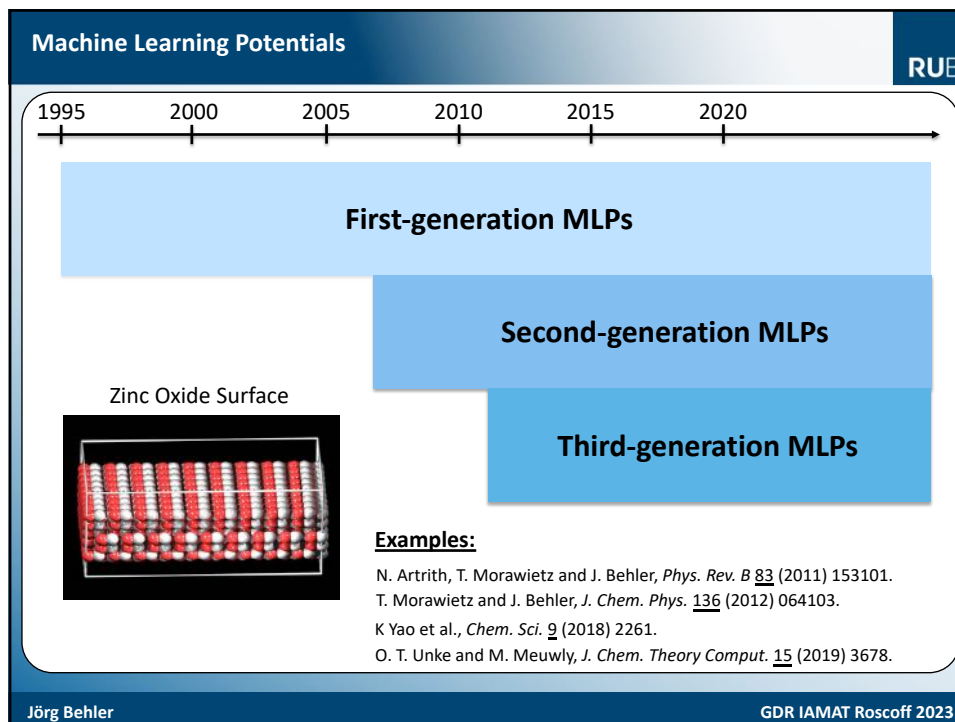
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Third-Generation Neural Network Potentials: Long-Range Interactions

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Forces RUB

Short-range part:

Force with respect to some coordinate α :

$$F_{\alpha,s} = -\frac{\partial E_s}{\partial \alpha} = -\sum_{j=1}^{N_{\text{atom}}} \frac{\partial E_j}{\partial \alpha}$$

$$\stackrel{\text{chain rule}}{=} -\sum_{j=1}^{N_{\text{atom}}} \sum_{\mu=1}^{N_{\text{sym},j}} \frac{\partial E_j}{\partial G_{j\mu}} \cdot \frac{\partial G_{j\mu}}{\partial \alpha}$$

from atomic NN architecture
from symmetry function definition

⇒ NNP provides exact analytic derivatives/forces

⇒ Important for energy conservation in MD

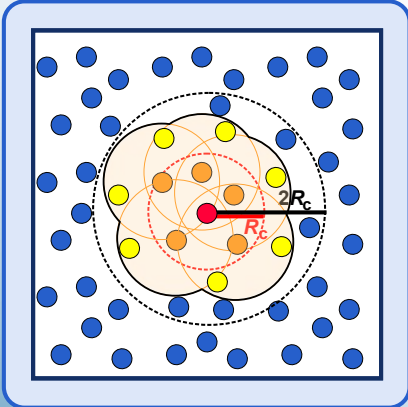
J. Behler, *J. Chem. Phys.* **134** (2011) 074106

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Environment-Dependence of the Forces RUB

An interesting consequence:
The environment-dependence of the forces is twice as large at the environment-dependence of the atomic energies!



J. Behler, Angew. Chem. Int. Ed. **56** (2017) 12828.

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Forces including electrostatics RUB

$$F_{\alpha} = F_{\alpha,s} + F_{\alpha,\text{elec}} = -\frac{\partial E_s}{\partial \alpha} - \frac{\partial E_{\text{elec}}}{\partial \alpha}$$

$$F_{\alpha,s} = -\frac{\partial E_s}{\partial \alpha} = -\sum_{j=1}^{N_{\text{atom}}} \frac{\partial E_j}{\partial \alpha} = -\sum_{j=1}^{N_{\text{atom}}} \sum_{\mu=1}^{N_{\text{sym},j}} \frac{\partial E_j}{\partial G_{j\mu}} \cdot \frac{\partial G_{j\mu}}{\partial \alpha}$$

$$F_{\alpha,\text{elec}} = -\frac{\partial}{\partial \alpha} \frac{1}{2} \sum_{i=1}^{N_{\text{atom}}} \sum_{j=1, j \neq i}^{N_{\text{atom}}} \frac{Q_i Q_j}{R_{ij}}$$

$$= -\sum_{i=1}^{N_{\text{atom}}} \sum_{j=1, j \neq i}^{N_{\text{atom}}} \frac{1}{2R_{ij}^2} \left[\underbrace{\frac{\partial Q_i}{\partial \alpha} Q_j R_{ij} + Q_i \frac{\partial Q_j}{\partial \alpha} R_{ij}}_{\text{not included in standard force fields}} - Q_i Q_j \frac{\partial R_{ij}}{\partial \alpha} \right]$$

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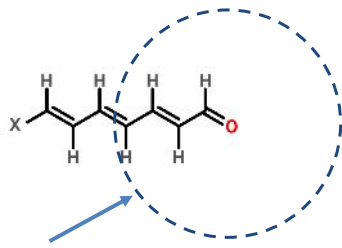
Fourth-Generation Neural Network Potentials: Global Electronic Structure

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Fourth-Generation Neural Network Potentials RUB

The Challenge:

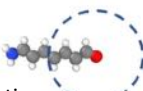
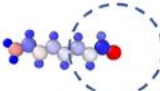
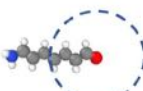
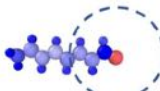
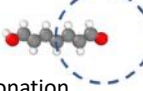
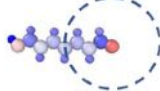
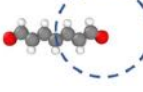
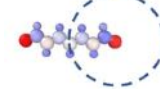
Example molecule:



cutoff radius

X = functional group

⇒ oxygen does not see the functional group X

	Structure	DFT charges
X=NH ₂		
protonation		
X=NH ₃ ⁺		
X=OH		
deprotonation		
X=O ⁻		

-0.25
+0.1
-0.4
+0.2

T. W. Ko, J. A. Finkler, S. Goedecker, J. Behler, Nat. Commun. 12 (2021) 398.

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Overview: Fourth-Generation Methods RUB

2015: Charge Equilibration Neural Network Technique - CENT

Idea: Global charge equilibration + Charge-dependent total energy expression
Applicability: ionic inorganic systems

S. A. Ghasemi, A. Hofstetter, S. Saha, S. Goedecker, Phys. Rev. B 92 (2015) 045131.

2020: Becke Population Neural Networks - BpopNN

Idea: SCF charge distribution, populations as additional inputs in modified SOAP
Applicability: molecular systems

X. Xie, K. A. Persson, D. W. Small, J. Chem. Theory Comput. 16 (2020) 4256.

2021: Fourth-Generation High-Dimensional NNPs – 4G-HDNNP

Idea: Global charge equilibration + accurate short-range energies
Applicability: very general

T. W. Ko, J. A. Finkler, S. Goedecker, J. Behler, Nat. Commun. 12 (2021) 398.

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CENT – Charge Equilibration Neural Network Technique RUB

$$E_{\text{tot}}(\{Q_i\}) = \sum_{i=1}^{N_{\text{atom}}} \left(E_i^0 + \chi_i Q_i + \frac{1}{2} J_{ii} Q_i^2 \right) + \frac{1}{2} \iint \frac{\rho(\mathbf{R}) \rho(\mathbf{R}')}{|\mathbf{R} - \mathbf{R}'|} d\mathbf{R} d\mathbf{R}'$$

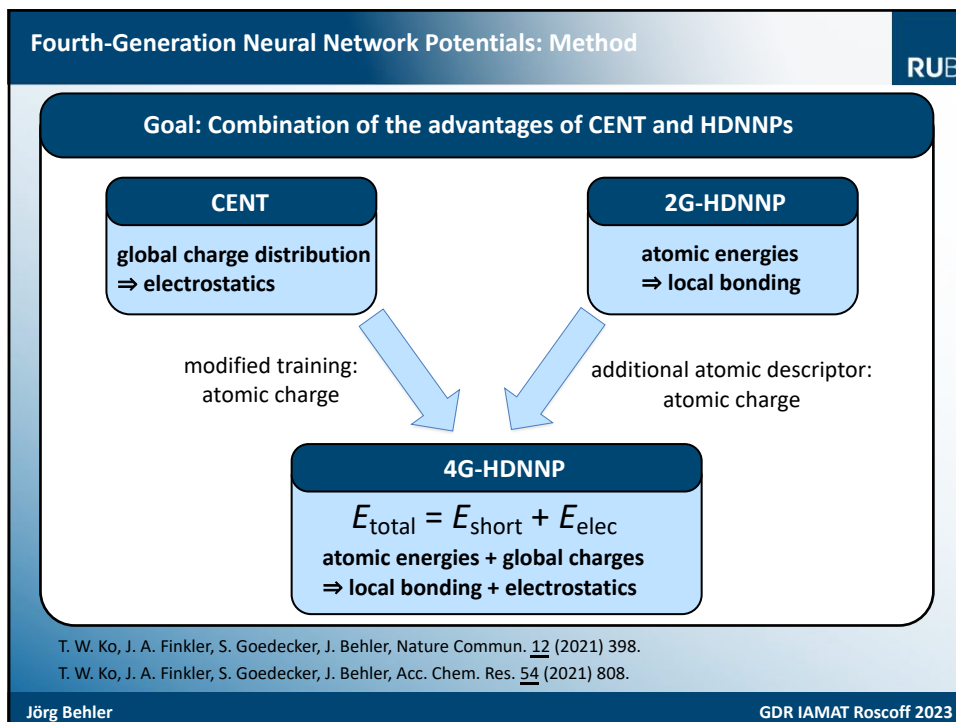
electronegativities
hardness
Coulomb energy

⇒ global electronic structure included (non-local charge transfer)
 ⇒ applications: systems with primarily ionic bonding

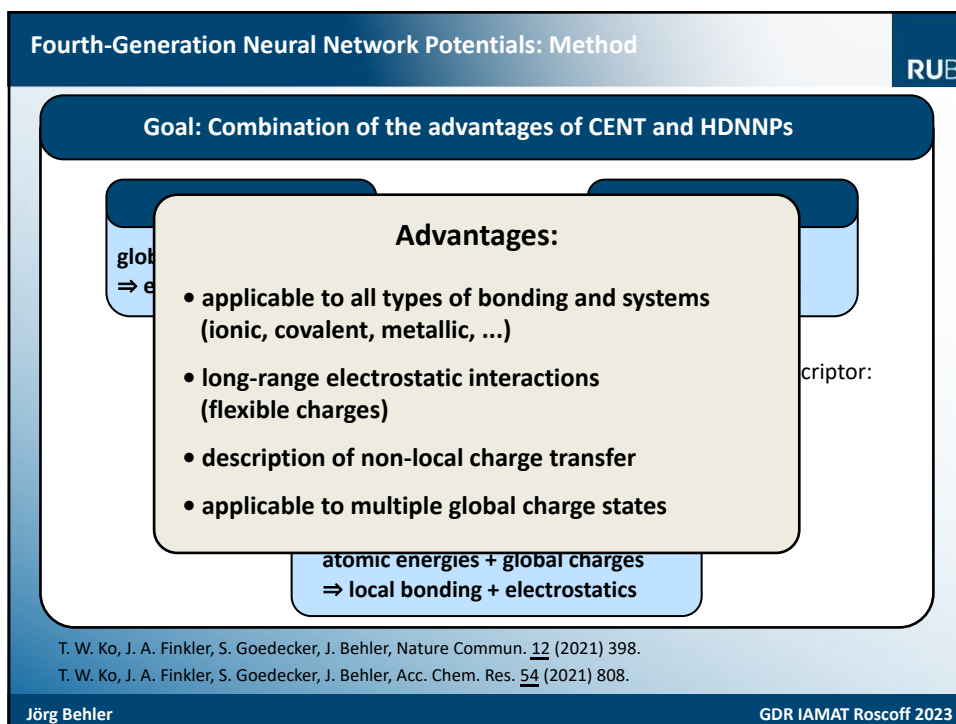
S. A. Ghasemi, A. Hofstetter, S. Saha, S. Goedecker, Phys. Rev. B 92 (2015) 045131.

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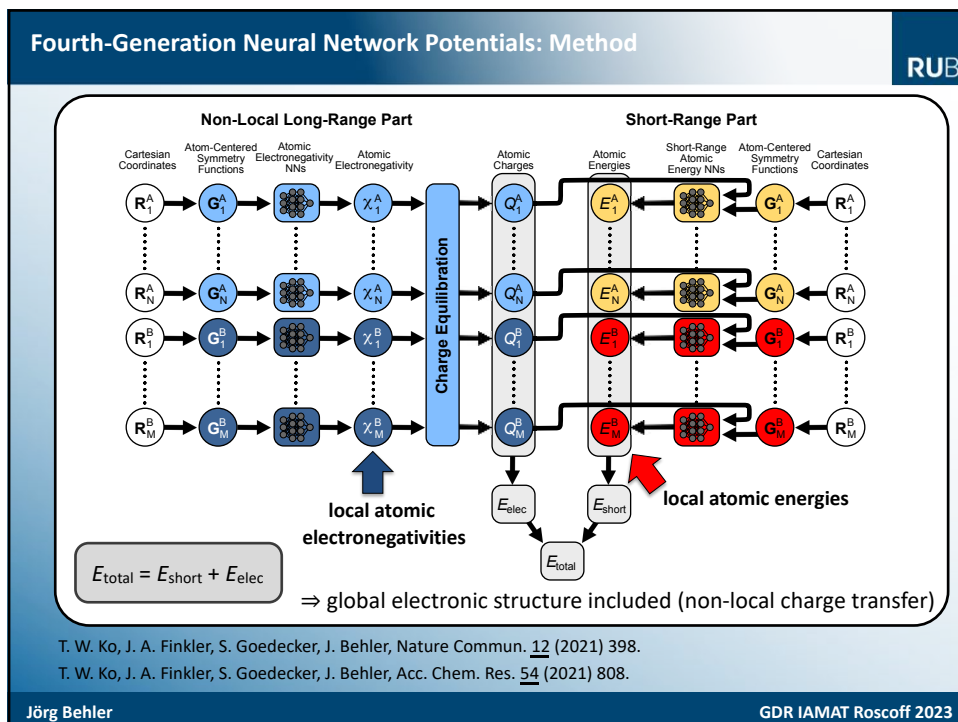
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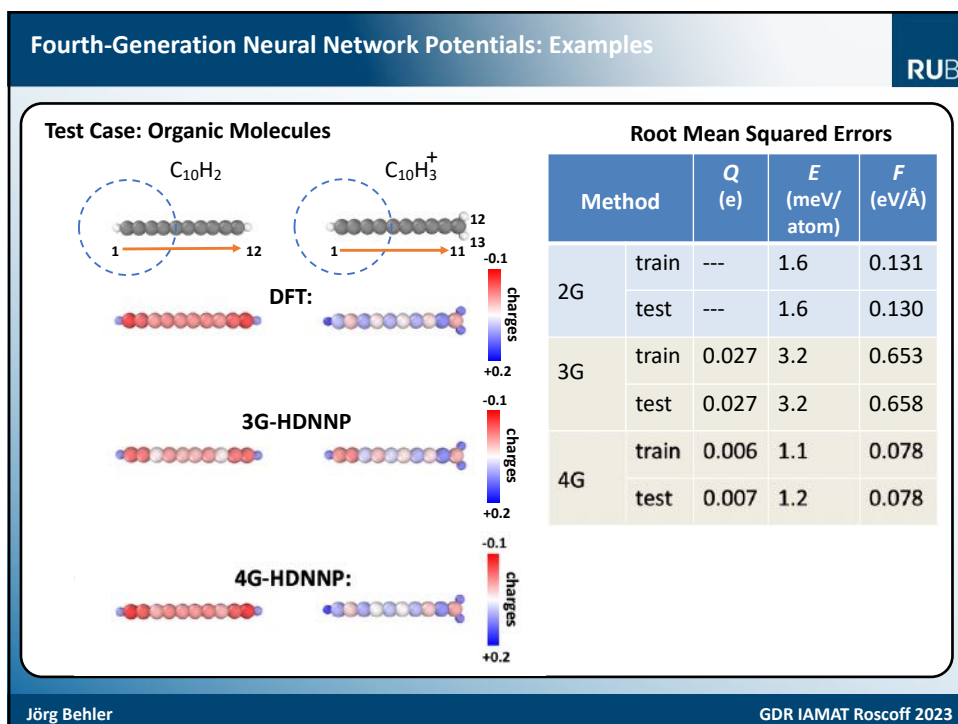
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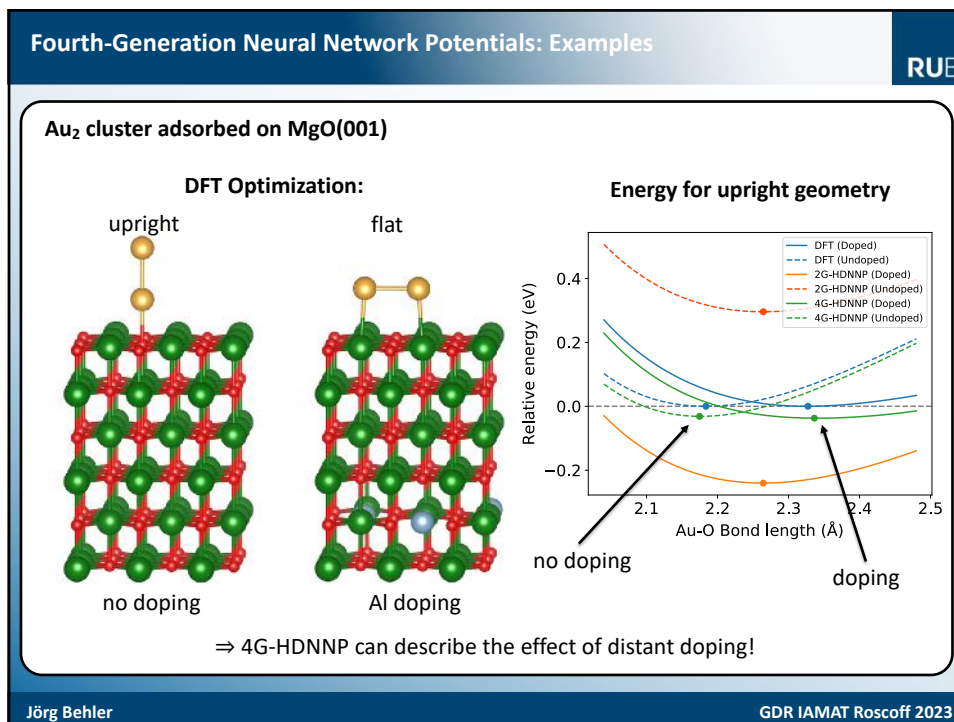
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Magnetic High-Dimensional Neural Network Potentials RUB

Most descriptors do not depend on atomic spins

antiferromagnetic

indistinguishable

ferromagnetic

⇒ spin-related energy changes are treated as noise
 ⇒ unreliable energy surfaces
 ⇒ **improved descriptors are needed**

M. Eckhoff and J. Behler, *npj Comput. Mater.* **7** (2021) 170.

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Magnetic High-Dimensional Neural Network Potentials RUB

Spin-Dependent Atom-Centered Symmetry Functions (sACSF)

Step 1: Atomic spin coordinates (collinear spin)

$$s_i = \begin{cases} 0 & \text{for } |M_S| < M_S^{\text{thres}} \\ \text{sgn}(M_S) & \text{otherwise} \end{cases} \quad \text{with} \quad M_S = \frac{1}{2}(n_{\uparrow} - n_{\downarrow})$$

Step 2: Spin-augmentation functions (SAF), radial case

$$M^0(s_i, s_j) = 1,$$

$$M^+(s_i, s_j) = \frac{1}{2} |s_i s_j| \cdot |s_i + s_j| \Rightarrow \text{spin-sensitive filter}$$

$$M^-(s_i, s_j) = \frac{1}{2} |s_i s_j| \cdot |s_i - s_j|$$

Step 3: sACSF, radial case

$$G_i^{\text{rad}} = \sum_j M^x(s_i, s_j) \cdot e^{-\eta R_{ij}^2} \cdot f_c(R_{ij})$$

⇒ **Second-generation (environment-dependent) extension**

⇒ Magnetic High-Dimensional Neural Network Potentials

M. Eckhoff and J. Behler, *npj Comput. Mater.* **7** (2021) 170.

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Quality Control

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Multistep Quality Control RUB

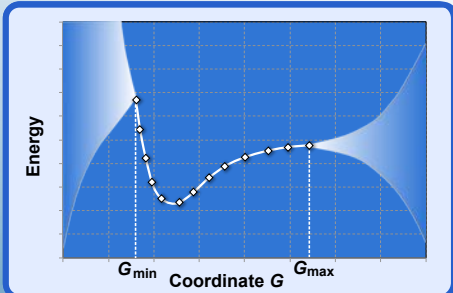
1. Generate reasonable initial training set
⇒ preliminary NN potential 
2. Check for extrapolation

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Multistep Quality Control RUB

1. Generate reasonable initial training set
⇒ preliminary NN potential 
2. Check for extrapolation




⇒ easy to detect

J. Behler, Angew. Chem. Int. Ed. 56 (2017) 12828.

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Multistep Quality Control RUB

1. Generate reasonable initial training set
⇒ preliminary NN potential 
2. Check for extrapolation
⇒ extend sampled "volume" in configuration space 
3. Use a validation set ("Early Stopping Method")

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Neural Network Potentials: Early Stopping Method RUB

Reference set → Training set: 90 % Generalization → Cross Validation Methods
 Reference set → Test set: 10 %

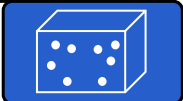
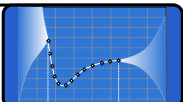
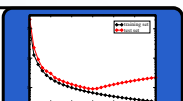
$$RMSE = \sqrt{\frac{1}{N} \sum_i (E_{i,NN} - E_{i,ref})^2}$$

⇒ overfitting cannot be detected by monitoring the error of the training set

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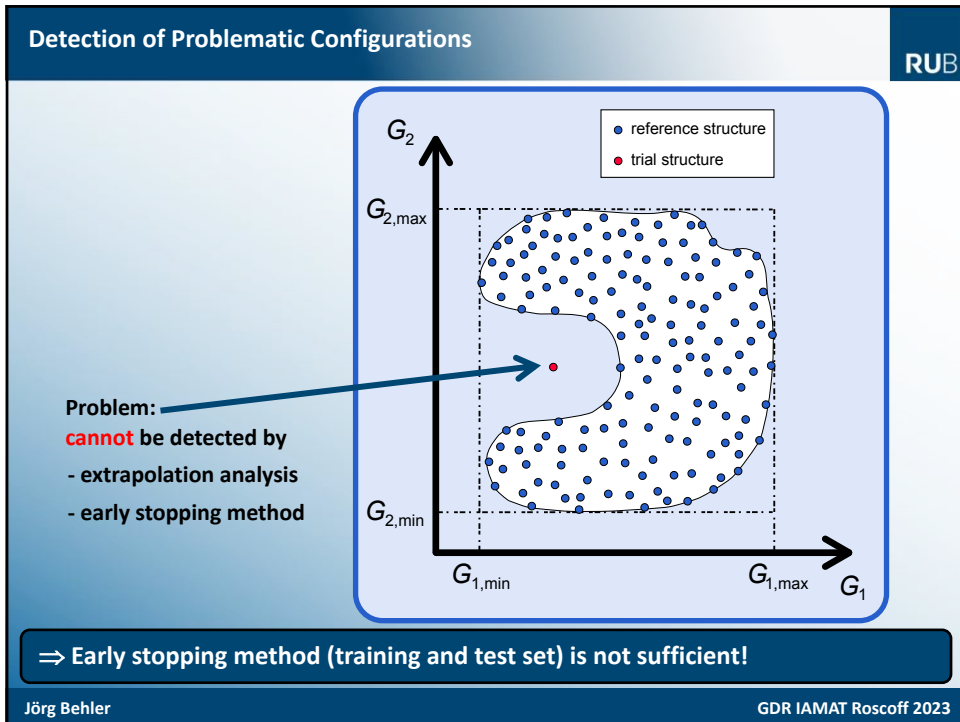
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Multistep Quality Control RUB

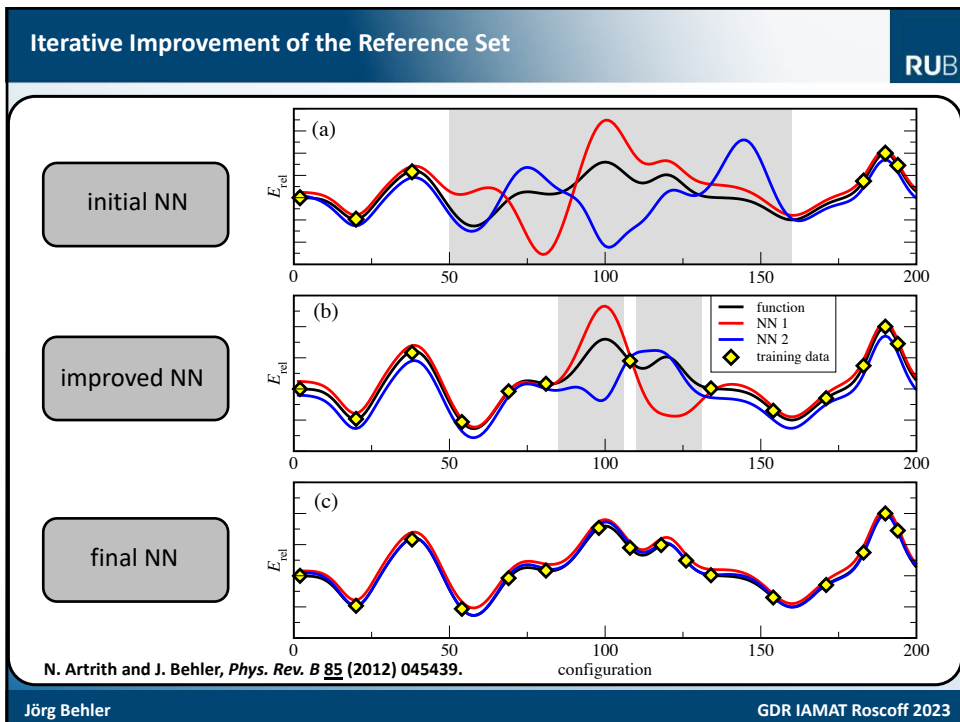
1. Generate reasonable initial training set
⇒ preliminary NN potential 
2. Check for extrapolation
⇒ extend sampled "volume" in configuration space 
3. Use a validation set ("Early Stopping Method")
⇒ estimate accuracy for similar structures not included in the training set 
4. Check for "holes in the training set"

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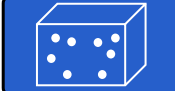
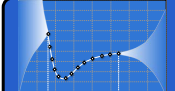
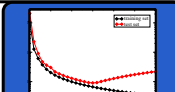

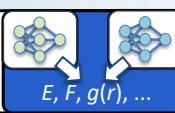
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Multistep Quality Control		RUB
1. Generate reasonable initial training set ⇒ preliminary NN potential		
2. Check for extrapolation ⇒ extend sampled "volume" in configuration space		
3. Use a validation set ("Early Stopping Method") ⇒ estimate accuracy for <u>similar</u> structures not included in the training set		
4. Check for "holes in the training set" ⇒ add structures in poorly sampled regions (iteratively)		
5. Check robustness of the results ⇒ repeat simulations with independent NN potentials	 $E, F, g(r), \dots$	
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Summary: Four Generations of Neural Network Potentials		RUB
First-Generation Neural Network Potentials		
Global description ⇒ low-dimensional systems		
Second-Generation Neural Network Potentials		
Short-range: yes	Long-range: no	Non-local: no
J. Behler, M. Parrinello, <i>Phys. Rev. Lett.</i> 98 (2007) 146401.		Spin: yes
Third-Generation Neural Network Potentials		
Short-range: yes	Long-range: yes	Non-local: no
N. Artrith, T. Morawietz and J. Behler, <i>Phys. Rev. B</i> 83 (2011) 153101.		Spin: yes
Fourth-Generation Neural Network Potentials		
Short-range: yes	Long-range: yes	Non-local: yes
T. W. Ko, J. A. Finkler, S. Goedecker, J. Behler, <i>Nature Commun.</i> 12 (2021) 398.		Spin: yes
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Summary: Four Generations of Neural Network Potentials RUB

First-Generation Neural Network Potentials

Global description \Rightarrow low-dimensional systems

Second-Generation Neural Network Potentials

Short-range: yes Long-range:

J. Behler, M. Parrinello, *Phys. Rev. Lett.* **98** (2007) 146401.

Third-Generation Neural Network Potentials

Short-range: yes Non-local: no Spin: yes

N. Artrith, J. Behler, *Phys. Rev. B* **83** (2011) 153101.

Fourth-Generation Neural Network Potentials

Short-range: yes Long-range: yes Non-local: yes Spin: yes

T. W. Ko, J. A. Finkler, S. Goedecker, J. Behler, *Nature Commun.* **12** (2021) 398.

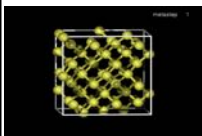
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Neural Network Potentials fill a gap
in the toolbox of chemistry, physics and materials science

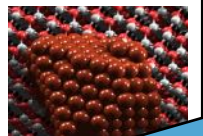
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Summary: Applications of Neural Network Potentials RUB

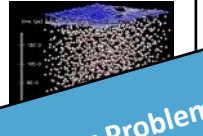
Bulk Materials


PRL **98** (2007) 146401.
PRL **100** (2008) 185501


Nanostructures


PRB **79** (2009) 115411.

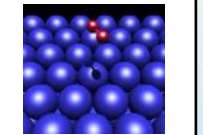
Nucleation


PNAS **108** (2011) 100103.

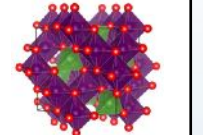
Phase Transitions


Nature Materials **10** (2011) 693.

Gas-Surface Dynamics


JPCL **10** (2019) 2957.
JPCL **10** (2019) 1763.

Li-Intercalation Compounds


PRB **102** (2020) 174102.
JCP **153** (2020) 164107.





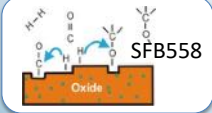











and many more ...

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Neural Networks Help Solving Problems

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